

Mirror Symmetry and the Type II String

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Abstract

If X and Y are a mirror pair of Calabi–Yau threefolds, mirror symmetry should extend to an isomorphism between the type IIA string theory compactified on X and the type IIB string theory compactified on Y , with all nonperturbative effects included. We study the implications which this proposal has for the structure of the semiclassical moduli spaces of the compactified type II theories. For the type IIB theory, the form taken by discrete shifts in the Ramond-Ramond scalars exhibits an unexpected dependence on the B -field. (Based on a talk at the Trieste Workshop on S-Duality and Mirror Symmetry.)

Mirror Symmetry and the Type II String

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1. INTRODUCTION

The dramatic recent progress in understanding nonperturbative aspects of string theory has come about through a study of various proposed equivalences between (perturbatively formulated) string theories. One such equivalence—one which has received relatively little attention in this regard—is mirror symmetry. Perhaps less attention has been paid because mirror symmetry is not a duality relating strong and weak string-couplings. However, the hypothesis that mirror symmetry extends to an equivalence between nonperturbative string theories has some interesting consequences for those theories [1], which we will review and extend here.

The equivalence which we consider relates the IIA string theory compactified to 4 dimensions on a Calabi–Yau manifold X , to the IIB theory compactified on the mirror partner Y of X .¹ (We shall refer to these compactified theories as IIA_X and IIB_Y , respectively.) One of the remarkable properties of mirror symmetry is the relationship which it establishes between the integer cohomology group $H^3(X, \mathbb{Z})$ and the “vertical” integer cohomology $\bigoplus H^{2k}(Y, \mathbb{Z})$ of the mirror partner [5–8]. This property is somewhat mysterious from the point of view of conformal field theory, since the integer cohomology plays no apparent rôle there.

In string theory, the integer cohomology groups

$H^3(X, \mathbb{Z})$ and $\bigoplus H^{2k}(Y, \mathbb{Z})$ find their proper rôle as likely candidates for describing the set of discrete shifts in the massless Ramond–Ramond scalars of the IIA_X and IIB_Y theories. The equivalence between the two can then be seen as a first step in establishing the equivalence between the full IIA_X and IIB_Y theories. However, there are some subtleties in the equivalence between integer cohomology groups which will lead us to the conclusion that the lattice of discrete shifts in Ramond–Ramond fields for a type IIB theory depends on the B -field as well as on $\bigoplus H^{2k}(Y, \mathbb{Z})$. This is somewhat reminiscent of the theta-angle dependence which occurs in Witten’s discussion of charge quantization of dyons [9].

2. MIRROR SYMMETRY IN STRING THEORY

Mirror symmetry [10–13] was originally formulated as a property of two-dimensional nonlinear sigma-models on Calabi–Yau manifolds. These sigma-models flow to $N = (2, 2)$ superconformal field theories in the infrared, and it is possible to find “mirror pairs” of such manifolds whose associated CFTs become isomorphic once the sign of the left-moving $U(1)$ -charge has been changed in one of the two theories. Mirror symmetry relates the CFT moduli spaces of the two Calabi–Yau manifolds, producing local isomorphisms between the Kähler moduli space of one manifold and the complex-structure moduli space of its mirror partner.

If a Calabi–Yau manifold X of dimension d is

¹The similarities between the IIA and IIB theories on X were studied some time ago [2], and the connection to mirror symmetry was pointed out in [3, 4, 1].

used to compactify the type IIA or IIB string, the $N = (2, 2)$ moduli space of X is embedded in the NS-NS sector of the moduli space of the effective $(10-d)$ -dimensional theory.² To see how a sign change in a worldsheet $U(1)$ -charge would affect the effective field theory, we consider Minkowski space $\mathbb{R}^{9-d,1}$, and work in light-cone gauge in which the spacetime fermions transform in spinor representations of $SO(8-d)$. The left-moving $\hat{u}(1)$ affine algebra from the $N = (2, 2)$ algebra of the superconformal field theory lies in the affine algebra $\hat{so}(8-d)$, and the weights of $SO(8-d)$ are charged under $U(1)$. A change of sign in left-moving worldsheet $U(1)$ -charge must therefore be accompanied by an action of the automorphism C of the weight space of $SO(8-d)$ which changes the signs of all of the weights.

If d is divisible by 4, then C maps each spinor representation of $SO(8-d)$ to itself. Thus, the worldsheet sign-change will leave the IIA theory as a IIA theory, and the IIB theory as a IIB theory. On the other hand, if d is congruent to 2 modulo 4, then C maps each spinor to the spinor of opposite chirality. It follows that in this case, an exchange between the IIA and IIB theories must accompany the worldsheet sign-change.

If X and Y are a mirror pair with d congruent to 2 modulo 4, then mirror symmetry should relate IIA_X to IIB_Y and vice versa. This equivalence can be considered as an analogue of the supersymmetric $R \leftrightarrow 1/R$ equivalence [14,15], which identifies IIA_{S^1} at large radius with IIB_{S^1} at small radius; there are extensions of this to compactifications on a torus of arbitrary dimension. In the case of $d = 2$, we have $X = Y = T^2$ and mirror symmetry can be derived from this $R \leftrightarrow 1/R$ equivalence. In the case of $d = 6$, however, mirror symmetry should provide a *further* equivalence between type II theories which is not a direct consequence of $R \leftrightarrow 1/R$, since most Calabi–Yau threefolds do not have an S^1 factor or even an action of S^1 .³

²The conformal field theory on X can also be regarded as an $N = (0, 2)$ theory and used to compactify the heterotic string, but as the implications of mirror symmetry are not as well-understood in this context we will focus on the type II string.

³There has been some recent speculation [16] about yet a third type of equivalence between type II theories, which

On the other hand, if X and Y are a mirror pair and d is divisible by 4, mirror symmetry relates IIA_X to IIA_Y and IIB_X to IIB_Y . The familiar cases of mirror symmetry in these dimensions are $X = Y = T^4$ and $X = Y = K3$, both with $d = 4$. Because these manifolds are self-mirror, mirror symmetry acts as a discrete identification on the moduli spaces. (In the $K3$ case, this has been used to establish [18] that the discrete identifications which act on the moduli space for IIA_{K3} are precisely the same as those of the moduli space for the heterotic string compactified on T^4 [19,20], as had been conjectured by Seiberg [21].) There may well be similar mirror identifications in $d = 8$, compactifying the type II string on one of Joyce’s manifolds [22] with holonomy $\text{Spin}(7)$.⁴

3. THE CONFORMAL FIELD THEORY MODULI SPACE

When passing from a classical to a quantum moduli space, new degrees of freedom may arise from the following mechanism. A continuous symmetry of the classical theory may be broken to a discrete symmetry of the quantum theory. There will be some massless scalar fields whose expectation values in the classical theory can be shifted to some fixed value (typically to zero) by exploiting the symmetry. In the quantum theory, however, the possible shifts are restricted to a discrete set, and the expectation values of these fields (modulo the discrete identifications) provide the new degrees of freedom. We shall use the term “semiclassical moduli space” to refer to the space obtained from the classical moduli space by including these new degrees of freedom. The discrete identifications are a quantum effect which must be respected by any further perturbative or nonperturbative quantum corrections to the moduli space.

A familiar example of this mechanism is provided by the nonlinear sigma-model on a Calabi–Yau manifold X . This model behaves at large radius like a field theory on X , and the “classical”

would relate IIA_{J^7} to IIB_{J^7} for compactifications on a Joyce manifold J^7 of holonomy G_2 [17].

⁴Some preliminary suggestions about mirror phenomena in this case were made in [23].

moduli space is the space of possible Ricci-flat metrics (normalized to have a fixed volume). The volume V , or radius $R = V^{1/d}$, of the manifold measures the size of quantum effects in the theory (which are suppressed at large radius). The new degree of freedom which arises in the quantum moduli space is the so-called B -field, which is a harmonic 2-form on X .⁵ It is well-defined only up to shifts $B \mapsto B + \delta B$, where δB is a 2-form which represents a class in *integer* cohomology, i.e., $\delta B \in H^2(X, \mathbb{Z})$. The fact that it is the integer cohomology which describes the discrete shifts follows from the presence of a “topological” term $S_{top} = \int_{\Sigma} \varphi^*(B)$ in the sigma-model action. (Here φ is a map from the worldsheet Σ to X). If the action is normalized so that $\exp(2\pi i S_{top})$ is what appears in physically measurable quantities, then the discrete symmetry of B must be represented by integer cohomology in order that $\int_{\Sigma} \varphi^*(\delta B)$ will be an integer, and hence that $\exp(2\pi i \int_{\Sigma} \varphi^*(\delta B))$ will be equal to 1.

The semiclassical description of the CFT moduli space is only valid at large radius, and indeed the structure of the moduli space at small radius is known to be substantially altered by nonperturbative effects (worldsheet instantons). Mirror symmetry (in conjunction with certain nonrenormalization theorems [28]) has been very useful in understanding the structure of the moduli space in these regions [29], since it relates regions of small radius on X to regions of large radius on the mirror partner Y . More precisely, the condition that the mirror partner Y be at large radius translates into a requirement that the complex structure on X lie in a certain region of the complex-structure moduli space (with no condition on the radius). The complex-structure moduli space is unaffected by the worldsheet instantons.

Consider now a path in the moduli space of Y in which the metric is fixed, and the B field takes the value $B_0 + t \delta B$, for $0 \leq t \leq 1$. Thanks to the discrete identification, this path forms a loop in the semiclassical moduli space. The mirror image of such a loop is a loop in the complex-

structure moduli space \mathcal{M}_X of X which encircles a boundary component of that moduli space. (Transporting structures along such loops will play an important part in our analysis below.) As the radius of Y increases, the corresponding loop γ in \mathcal{M}_X shrinks towards the boundary component. In fact, if we rescale the metric on Y by $g_{ij} \mapsto \lambda g_{ij}$ and let $\lambda \rightarrow \infty$, then the loops $\gamma(\lambda)$ will sweep out a punctured disk whose limit point is on the boundary component (cf. [30]). For an appropriate compactification $\overline{\mathcal{M}}_X$ of the complex-structure moduli space \mathcal{M}_X , the mirror of a large radius limit point appears as an intersection of $r = \dim(\mathcal{M}_X)$ boundary components, and the disks swept out by $\gamma_1(\lambda), \dots, \gamma_r(\lambda)$ provide local complex coordinates t_1, \dots, t_r on the compactification.

4. THE TYPE IIA MODULI SPACE

We specialize now to the case $d = 6$, so that X is a Calabi–Yau threefold. We assume that the first Betti number of X is 0, or equivalently, that Ricci-flat metrics on X have holonomy precisely $SU(3)$. If we compactify the IIA string on X , the massless scalar spectrum of the theory consists of the metric, the B -field, the axion θ and the dilaton ϕ in the NS-NS sector, and a field which corresponds to a harmonic 3-form C in the R-R sector.⁶ The “classical” moduli space for the IIA $_X$ theory coincides with the conformal field theory moduli space described above—at large radius, it is accurately described by the metric and B -field, modulo discrete shifts of the B -field. The dilaton measures the size of quantum effects in the IIA theory, and the other new fields—the axion and the R-R 3-form C —should be subject to discrete symmetries as we have discussed. In the case of the axion, this is well-known: the shift is by integer multiples of 2π , and it is common to use a parameter $\exp(8\pi^2 S) = \exp(-i\theta + 8\pi^2 e^{-2\phi})$ which combines the axion and dilaton and also implements the identifications by discrete shifts. However, in the case of the 3-form, the precise nature of the discrete shift is more difficult to pin down.

⁵We ignore the additional degree of freedom which is provided by torsion in $H_2(X)$ [24–27].

⁶We again ignore any effects that may be associated with torsion in homology.

If an action for 2-branes plays a rôle in the eventual nonperturbative formulation of type IIA string theory, then a topological term in that action of the form $S_{top} = \int_M \psi^*(C)$ would cause the discrete shift to take the form $C \mapsto C + \delta C$ with $\delta C \in H^3(X, \mathbb{Z})$. (Here ψ is a map from the world-volume M to the target space X , and the argument is completely analogous to the case of the B -field.) However, it would be preferable to arrive at this conclusion without assuming such details about the form of the nonperturbative theory. In [1] it was argued that for many Calabi–Yau threefolds, mirror symmetry combined with S -duality for the type IIB theory implies that the discrete shifts δC must fill out a finite index subgroup of $H^3(X, \mathbb{Z})$.

We shall assume (for simplicity) that $H^3(X, \mathbb{Z})$ provides the correct set of discrete shifts for the quantum theory.⁷ We shall also assume that X has a mirror partner Y . Then the semiclassical moduli space of the IIA _{X} theory has the following description. The CFT moduli space is essentially a product of the complex-structure moduli spaces \mathcal{M}_X and \mathcal{M}_Y of X and Y . (There are some subtleties about that statement, but they need not concern us here.) The R-R 3-form C will transform in a vector bundle $\mathcal{E}_X \rightarrow \mathcal{M}_X$ whose fibers are the cohomology groups $H^3(X, \mathbb{R})$. Within that bundle is a bundle of lattices $\mathcal{E}_X^{\mathbb{Z}}$ which describe the discrete identifications (the fibers of which are $H^3(X, \mathbb{Z})$). The complex-structure moduli and C together fill out the quotient $\mathcal{E}_X/\mathcal{E}_X^{\mathbb{Z}}$, a bundle of tori. Finally, the axion-dilaton field $\exp(8\pi^2 S)$ transforms in a \mathbb{C}^* -bundle $\mathcal{L}^* \rightarrow (\mathcal{E}_X/\mathcal{E}_X^{\mathbb{Z}})$, and the entire semiclassical moduli space can be described as $\mathcal{L}^* \times \mathcal{M}_Y$ (up to the subtlety about the product structure alluded to earlier).

The bundle of tori $(\mathcal{E}_X/\mathcal{E}_X^{\mathbb{Z}}) \rightarrow \mathcal{M}_X$ is a familiar object in algebraic geometry, called the family of *intermediate Jacobians*. These tori come equipped with natural complex structures by means of the isomorphisms

$$H^3(X, \mathbb{R}) \cong H^{3,0}(X_t) \oplus H^{2,1}(X_t) \quad (1)$$

⁷The bulk of our analysis could be restated (in a more cumbersome fashion) for the case of a finite index subgroup.

for $t \in \mathcal{M}_X$. Griffiths [31] proved that these complex structures vary holomorphically, that is, the total space $\mathcal{E}_X/\mathcal{E}_X^{\mathbb{Z}}$ is itself a complex manifold and the map to \mathcal{M}_X is holomorphic. Donagi and Markman [32] have recently shown that $\mathcal{E}_X/\mathcal{E}_X^{\mathbb{Z}}$ has the additional structure of being a complex contact manifold, and that the \mathbb{C}^* -bundle $\tilde{\mathcal{L}}^* \rightarrow \mathcal{E}_X/\mathcal{E}_X^{\mathbb{Z}}$ whose fibers are the nonzero elements of $H^{3,0}(X_t)$ is the associated complex symplectic manifold. This is precisely the geometry that one expects will underlie a hyper-Kähler metric. (The non-compactness of $\tilde{\mathcal{L}}^*$ prevents us from immediately concluding that such a metric exists.) It is tempting to identify $\tilde{\mathcal{L}}^*$ with \mathcal{L}^* since a quaternionic Kähler metric is expected on the latter, of which the putative hyper-Kähler metric is perhaps a limit. It should be possible to settle this question and determine the precise nature of the metric on \mathcal{L}^* by means of the explicit “c-map” of [33]. This issue is currently under investigation.

We can expect nonperturbative corrections to the semiclassical moduli space of various kinds. A mirror version of the conifold transitions of [34] should link together some (perhaps all) of these moduli spaces. Other nonperturbative effects should modify the structure of this space at strong coupling [35].

5. MONODROMY

The description we have given of the semiclassical type IIA moduli space was complicated by the unavoidable fact that the bundle of lattices $\mathcal{E}_X^{\mathbb{Z}}$ is *not* trivial over \mathcal{M}_X . If we follow the $H^3(X_t, \mathbb{Z})$ lattice as t traverses a loop γ in \mathcal{M}_X , then the lattice undergoes a *monodromy transformation* represented by a matrix T_γ . In particular, it is not usually possible to find a single-valued function $t \mapsto \Gamma(t) \in H^3(X_t, \mathbb{Z})$ as t traverses such a loop.

The behavior of these monodromy matrices is not arbitrary, however. By the monodromy theorem [36], all eigenvalues of T_γ are roots of unity. Furthermore, if we work near the mirror of a large radius limit point, using loops $\gamma_1, \dots, \gamma_r$ as at the end of section 3, then the eigenvalues are all 1 and the logarithms $N_j := \log T_{\gamma_j}$ are a commuting set of nilpotent matrices. The nilpotent orbit theorem [37] says that although a function of the form

$t \mapsto \Gamma(t)$ taking values in $H^3(X_t, \mathbb{Z})$ is not single-valued, the function

$$t \mapsto \exp\left(-\sum \frac{\log t_j}{2\pi i} N_j\right) \Gamma(t) \quad (2)$$

is single-valued and behaves well at the boundary, where t_1, \dots, t_r are the coordinates associated to $\gamma_1, \dots, \gamma_r$.

(We wish to stress that this same result would have been obtained for *any* lattice within $H^3(X_t, \mathbb{R})$; it does not depend on our assumption that the discrete shifts correspond to integer cohomology.)

The mirror of the monodromy transformations $\exp(N_j)$ have a very natural description. Traversing the loop γ_j in \mathcal{M}_X corresponds to following a path $B = B_0 + t(\delta B)_j$, $0 \leq t \leq 1$ in the NS-NS moduli space of Y , where $(\delta B)_j \in H^2(Y, \mathbb{Z})$. The action of the monodromy on $H^3(X, \mathbb{R})$ is then mapped to an action of the B -field shift $(\delta B)_j$ on the vertical cohomology $H^{even}(Y, \mathbb{R})$ described by

$$C \mapsto \exp((\delta B)_j) \wedge C \in H^{even}(Y, \mathbb{R}), \quad (3)$$

where we write $C = \sum_{k=0}^3 (3-k)! C_{2k}$, and $\exp(B) = 1 + B + \frac{1}{2}(B \wedge B) + \frac{1}{6}(B \wedge B \wedge B)$, both regarded as elements of $H^{even}(Y, \mathbb{R})$. The factor of $(3-k)!$ is designed to make the integer cohomology work out appropriately, and indeed if $\delta B \in H^2(Y, \mathbb{Z})$ then $\exp(\delta B)$ will map $\bigoplus_{k=1}^3 (3-k)! H^{2k}(Y, \mathbb{Z})$ to itself.

In all known examples of mirror pairs for which the integer cohomology has been computed, that cohomology exhibits the following beautiful structure (see [8] for a review): $H^3(X, \mathbb{Z})$ is mapped to $H^{even}(Y, \mathbb{Z})$ in such a way that the monodromies $\exp(N_j)$ are sent to the mappings of eq. (3). (Both of these transformations are well-defined on integer cohomology.) It is conjectured that this is always the case.

6. THE TYPE IIB MODULI SPACE

We continue to let Y denote a Calabi–Yau threefold with $b_1(Y) = 0$ which has X as its mirror partner. The semiclassical moduli space for the type IIB_Y theory is more difficult to describe than that for the type IIA theory, in part because we lack a Lagrangian formulation for type IIB

theories. The NS-NS sector of the massless spectrum still consists of the metric, the B -field, the axion and the dilaton; however, the R-R sector is harder to identify. The R-R field content of the IIB theory in 10 dimensions consists of a 0-form, a 2-form and a self-dual 4-form, which one might expect to give rise by dimensional reduction to two harmonic 0-forms, two harmonic 2-forms and a harmonic 4-form. However, the self-duality condition on the field strength of the 4-form reduces the R-R degrees of freedom to two 0-forms, one 2-form and one 4-form. We shall replace one of the 0-forms by its Hodge star (a 6-form), leaving us with R-R fields C_{2k} : a harmonic $2k$ -form for each of $k = 0, 1, 2, 3$. This replacement is motivated by the way in which mirror symmetry identifies $H^3(X)$ with $H^{even}(Y)$. In the absence of a Lagrangian our identification of the R-R fields is somewhat tentative, but we have at least gotten the number of degrees of freedom right.

The semiclassical moduli space for the type IIB_Y theory can now be built up as before. We start with the CFT moduli space $\mathcal{M}_X \times \mathcal{M}_Y$, include the axion-dilaton field as transforming in a \mathbb{C}^* -bundle \mathcal{L}^* , and describe the R-R fields as taking values in a bundle over the NS-NS moduli space with fibers $H^{even}(Y, \mathbb{R})/\Lambda$, where Λ represents the discrete shifts. As in the type IIA case, we expect nonperturbative corrections—both at weak coupling due to conifold transitions [34], and at strong coupling.

The semiclassical moduli spaces of IIA_X and IIB_Y cannot be globally isomorphic, for a simple reason: the R-R fields in the IIB case are modeled on $H^{even}(Y)$, which in conformal field theory was subject to worldsheet instanton corrections, whereas the R-R fields in IIA are modeled on $H^3(X)$ which has no such corrections. It is expected that the worldsheet instanton corrections will be mimicked by nonperturbative solitons in the IIB theory [35] which will correct the moduli space at strong coupling. In any event, any proposed mirror isomorphism between IIA_X and IIB_Y semiclassical moduli spaces should only be considered locally, near weak coupling. We can still reliably study the discrete shifts there.

We have not yet determined the discrete identifications Λ of the R-R fields. At first glance

it would appear that the most natural guess for this lattice would be the vertical integer cohomology $H^{even}(Y, \mathbb{Z})$. If mirror symmetry is to hold, however, this guess cannot be correct. For if it were, the semiclassical moduli space would be a *trivial* bundle over the NS-NS moduli space, with fiber $H^{even}(Y, \mathbb{R})/H^{even}(Y, \mathbb{Z})$. That is not the structure we found on the type IIA side, since the monodromy is missing. The only alternative is that *the precise values of the possible discrete shifts δC_{2k} must depend on the value of B !*

As we saw in the previous section, upon shifting the B -field by $\delta B \in H^2(Y, \mathbb{Z})$, the vertical integer cohomology classes will shift by wedging with $\exp(\delta B)$. The most straightforward way to reproduce this monodromy behavior is to postulate the following structure for the discrete shifts.⁸ Let $C = \sum_{k=0}^3 (3-k)! C_{2k}$, and let δC be the discrete shift. The condition we should require is:

$$\exp(B) \wedge (\delta C) \in \bigoplus_{k=0}^3 (k!) H^{6-2k}(Y, \mathbb{Z}). \quad (4)$$

It is worthwhile to write this out more explicitly:

$$\begin{aligned} \delta C_0 &\in H^0(Y, \mathbb{Z}) \\ \delta C_2 + 3B \wedge \delta C_0 &\in H^2(Y, \mathbb{Z}) \\ \delta C_4 + 2B \wedge \delta C_2 + 3B \wedge B \wedge \delta C_0 &\in H^4(Y, \mathbb{Z}) \\ \delta C_6 + B \wedge \delta C_4 + B \wedge B \wedge \delta C_2 \\ &+ B \wedge B \wedge B \wedge \delta C_0 \in H^6(Y, \mathbb{Z}). \end{aligned} \quad (5)$$

Our condition has the property that when we shift B to $B + \delta B$ then the condition changes by wedging with $\exp(\delta B)$, precisely reproducing the anticipated monodromy effect. Of course it is also possible to imagine a more complicated, nonlinear B -field dependence with the same property.

What kind of nonperturbative effect in type IIB theory could produce such a condition? We would appear to need topological terms in actions for p -branes for each of $p = -1, 1, 3, 5$. For example, the third condition might follow from a 3-brane action with a topological term of the form

$$S_{top} = \int_{M^4} \eta^*(C_4 + 2B \wedge C_2 + 3B \wedge B \wedge C_0) \quad (6)$$

⁸In [1], we described this structure in terms of the quantum cohomology ring of Y . The two formulations are essentially equivalent.

(η being a map from the worldvolume to the target space). Why the term should take precisely this form is somewhat mysterious.

It is tempting to speculate that a related B -field dependence will appear in the charge quantization rules for R-R gauge fields, similar to Witten's discussion of charge quantization for dyons in the presence of a theta-angle [9].

7. CONCLUSIONS

Let us summarize our analysis in the following way. There are three conjectures which seem very reasonable, and which reinforce each other nicely. The first is that the discrete shifts of the R-R fields in the IIA_X theory are given by $H^3(X, \mathbb{Z})$ when $b_1(X) = 0$.⁹ The second is that the integer cohomology and integer monodromies are preserved by mirror symmetry. (There is concrete evidence for this second conjecture.) And the third is that the discrete shifts of the R-R fields in the IIB_Y theory are given by eq. (5). Together, these conjectures suggest a coherent picture of mirror symmetry in string theory, giving a local isomorphism between the semiclassical moduli spaces at weak coupling. It is to be hoped that the isomorphism will extend to the full moduli spaces once nonperturbative effects are taken into account.

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⁹If $b_1(X) \neq 0$, we should actually expect some dependence on the B -field in this case as well, through terms such as $B \wedge C_1$. This follows from the analysis of $K3 \times T^2$ given in [1].

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